International Journal of Modern Physics A, Vol. 4, No. 3 (1989) 737–752


# ACTION PRINCIPLE FOR OVERDETERMINED SYSTEMS OF NONLINEAR FIELD EQUATIONS

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Received 27 May 1988

We propose a general scheme for constructing an action principle for arbitrary consistent overdetermined systems of nonlinear field equations. The principal tool is the BFV-BRST formalism. There is no need for star-product nor Chern-Simons forms. The main application of this general construction is the derivation of a superspace action in terms of unconstrained superfields for the $D = 10$ $N = 1$ Super-Yang-Mills theory. The latter contains cubic as well as quartic interactions.

## 1. Motivation

Supersymmetric gauge theories are formulated in terms of geometrical constraints amounting to vanishing of some curvatures of the superfield gauge potentials $A^{\mu}(x,\theta)$, $A^{\alpha}(x,\theta)$ along certain hyperplanes in superspace.[1,2,3,4] For $D = 4$ $N = 3{,}4$ and $D = 10$ $N = 1$[5] supersymmetric Yang-Mills theories (SYM) these constraints are actually equivalent to the equations of motion[2,6,7,8,9] (unlike the $D = 4$ $N = 1, 2$ case).

Due to no-go theorems,[10] it is impossible to find an off-shell action principle in terms of ordinary (extended) superfields (except for $D = 4$ $N = 1$) in which to produce the above curvature constraints as variation equations. The no-go theorems were successfully circumvented in the case of $D = 4$, $N = 2, 3$ SYM[11] by an appropriate extension of the notion of superspace (harmonic superspace) and after a suitable reformulation of the curvature constraints with the help of the auxiliary harmonic variables. However, the attempts to generalize this procedure to the $N = 4$ $D = 4$ and, correspondingly, $N = 1$ $D = 10$ SYM proved unsuccessful so far (cf. Ref. 12).

From mathematical point of view the curvature constraint equations together with their consequences implied by the Biancchi identities constitute a consistent overdetermined set of nonlinear field equations for the supergauge potentials $A^{\mu}(x,\theta)$, $A^{\alpha}(x,\theta)$. Here "overdetermined" means that the number of equations is greater than the number of unknown functions, and "consistent" (or integrable) means that the overdetermined system possesses nontrivial solutions.

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Similar structures arise in the covariant second quantization of constrained Hamiltonian systems. Indeed, we have $\mathcal{N}$ (= number of Dirac first class constraints) linear (matrix) constraint equations for one (vector-valued) wave function, i.e. $\mathcal{N}$ classical (matrix) free-field equations for one classical (vector-valued) field. In the interacting theory we would get $\mathcal{N}$ nonlinear (matrix) field equations constituting a consistent overdetermined system (cf. (2.1)–(2.4) below).

In string theory, one is also confined in the position of "guessing" the (superstring-) field action principle corresponding to the known equations of motion.

Our primary aim in this paper is to present a general construction (Sec. 2) of an off-shell action principle for such nonlinear systems. The main tool is the BFV-BRST ghost formalism.[13] The action resembles the Siegel-Zwiebach-Witten-Neveu-West[14] construction of (super)string field actions but does not involve the peculiarities (star products, Chern-Simons forms etc.) specific to the field theory of the Ramond-Neveu-Schwarz (RNS) (super)string.

The main application presented here (Sec. 3) is the construction of a superspace action for $D = 10$ $N = 1$ SYM in terms of unconstrained (off-shell) superfields. This action contains both cubic and quartic interaction terms. Besides depending on the ordinary superspace coordinates $(x^\mu, \theta_\alpha)$, the corresponding superfields also depend on the auxiliary (harmonic-like) bosonic variables $(u, v)$ (Eq. (3.8) below)[15–26] and on a number of BFV-BRST ghost variables $\eta^A$ (see Eqs. (3.38), (3.35) and (3.37) below). Thus, these generalized superfields contain an infinite number of pure gauge and auxiliary fields which are eliminated through the Witten-type nonlinear gauge invariance (Eq. (2.14) below) and through the usual superspace YM gauge invariance (Eq. (3.37) below) of our superspace action.

Let us particularly stress that, in our formalism, the YM gauge invariance is not a part of the Witten-type gauge invariance but it is an independent symmetry of our action. This phenomenon is most easily understood in the context of the heterotic GS superstring. Already its zero-mode (point-particle) limit contains the gauge invariant SYM whereas in the RNS formalism, the YM gauge invariance arises from the Witten's gauge invariance at the first excited string level in the NS sector.

## 2. The General Construction

Let us consider the following general overdetermined system of $\mathcal{N}$ nonlinear equations:

$$\mathcal{L}_A(\phi|z) \equiv L_A\phi(z) + V_A(\phi|z) = 0, \qquad A = 1, \ldots, \mathcal{N} \tag{2.1}$$

$$V_A(\phi|z) \equiv \sum_{n\geq 0} \int dz_1 \ldots dz_{n+2}\, V_A^{(n+2)}(z; z_1, \ldots, z_{n+2})\phi(z_1)\ldots\phi(z_{n+2}). \tag{2.2}$$

In (2.1), the function $\phi(z)$ is defined on a (graded) linear space $\mathcal{R}$ and it takes values in another (graded) vector space $\mathcal{U}$, i.e. has a vector index $\phi = (\phi^a(z))$. $L_A$ are (graded) linear operators with at most second order derivatives and are, correspondingly, matrices $(L_A \equiv (L_A^{ab}))$ in the vector space $\mathscr{U}$. Clearly, $V_A(\phi|z) = ([V_A(\phi|z)]^a)$ are also vectors in $\mathscr{U}$. In this section, the vector indices $a$, $b$ will be suppressed for brevity.

The necessary conditions for consistency of the overdetermined system (2.1) are obtained by multiple application of antisymmetrized products of the linear operators $L_B$ on $\mathscr{L}_A(\phi|z)$ (2.1) and by requiring the result to vanish when Eq. (2.1) is fulfilled. The first consistency condition

$$L_A \mathscr{L}_B(\phi|z) + (-1)^{\varepsilon_A\varepsilon_B+1} L_B \mathscr{L}_A(\phi|z) = 0$$

yields for the linear and nonlinear parts respectively:

$$[L_A, L_B\} \equiv L_A L_B + (-1)^{\varepsilon_A\varepsilon_B+1} L_B L_A = f_{AB}^C L_C \tag{2.3}$$

$$L_A V_B(\phi|z) + (-1)^{\varepsilon_A\varepsilon_B+1} L_B V_A(\phi|z) - f_{AB}^C V_C(\phi|z)$$
$$= \int dz' \left[ \frac{\delta V_B(\phi|z)}{\delta\phi(z')} \mathscr{L}_A(\phi|z') + (-1)^{\varepsilon_A\varepsilon_B+1} \frac{\delta V_A(\phi|z)}{\delta\phi(z')} \mathscr{L}_B(\phi|z') \right]$$
$$\big(= 0 \text{ on the surface of Eq. (2.1)}\big). \tag{2.4}$$

In (2.3), (2.4) $f_{AB}^C$ are in general linear operators and $\varepsilon_A$, $\varepsilon_B$ are the Grassmann parities of $L_A$, $L_B$ correspondingly. In Eq. (2.4) the operators $L_A$ act on $V_B(\phi|z)$ defined in Eq. (2.2) as on functions of $z$.

The next consistency condition

$$[L_C(-1)^{\varepsilon_B+\varepsilon_C} L_A V_B(\phi|z)]_{\text{antisymm}(A,B,C)} = 0 \text{ on the surface of Eq. (2.1)}$$

gives using (2.3), (2.4):

$$[f_{ABC}^{(2)\,DE}(-1)^{\varepsilon_D} f_{AD}^G]_{\text{antisymm}(A,B,C)} V_G(\phi|z) = 0$$

where the operator $f_{ABC}^{(2)\,DE}$ is defined by:

$$f_{ABC}^{(2)\,DE} L_E = \big((-1)^{\varepsilon_D+\varepsilon_B+1}\{(-1)^{\varepsilon_D\varepsilon_C}[f_{AB}^D, L_C] + f_{AB}^G f_{GC}^D\}\big)_{\text{antisymm}(ABC)} \tag{2.5}$$

and antisymmetrization means:

$$M_{\dots BA \dots} = (-1)^{(\varepsilon_A+1)(\varepsilon_B+1)} M_{\dots AB \dots}\,.$$

For most interesting systems it turns out that:

$$f_{ABC}^{(2)\,DE} = 0\,. \tag{2.6}$$

In what follows we accept (2.6) as fulfilled, which means that the only nontrivial consistency conditions for the system (2.1) are given by (2.3), (2.4). Let us immediately note that if the set of operators $L_A$ is viewed as a first quantized system of Dirac first class Hamiltonian constraints (cf. (2.3)), then $f^{(2)\,DE}_{ABC}$ defined by (2.5) is precisely the so-called second order BFV structure function.[13] Its vanishing (2.6) means that the corresponding Hamiltonian system is first rank, i.e. the corresponding BRST charge does not possess higher order ghost terms.

Our general construction of an action principle for the system (2.1) works under the following general assumptions:

(i) The number $N_b$ of bosonic operators $L_A$ in (1) (i.e. with $\varepsilon_A = 0$) has to be odd;

(ii) The linear operators $L_A$ must be functionally independent.

Since the system (2.1) comprises $\mathcal{N} = N_b + N_f > 1$ matrix equations it is impossible to find an action functional $S = S[\phi]$, depending on $\phi(z)$ alone, such that (2.1) would arise as equations of motion $\dfrac{\delta S}{\delta\phi(z)} = 0$.

Let us outline here the general steps of our construction:

*First step*

We have first to re-write the overdetermined set (2.1) of $\mathcal{N}$ (matrix) equations as a single (matrix) equation in terms of a (vector valued) field $\Phi(z,\eta)$ depending on auxiliary variables collectively denoted by $\eta$. The original field $\phi(z)$ from (2.1) enters as:

$$\Phi(z,\eta) = \phi(z) + \tilde{\Phi}(z,\eta)$$
$$\tilde{\Phi}(z,\eta) = \sum_{n\geq 1}\frac{1}{n!}\eta^{A_1}\ldots\eta^{A_n}\tilde{\phi}_{A_1\ldots A_n}(z). \tag{2.7}$$

To this end we take:

$$\eta = (\eta^A) = (c^i, \chi^\alpha) \qquad i = 1,\ldots,N_b, \qquad \alpha = 1,\ldots,N_f, \qquad A = 1,\ldots,\mathcal{N} = N_f + N_b \tag{2.8}$$

to be ghost variables associated with $L_A$, i.e. having opposite Grassmann parity $\varepsilon(\eta^A) = \varepsilon_A + 1$.

The new single (matrix) equation for $\Phi(z,\eta)$ replacing the system (2.1) will be of the following general form:

$$Q(\Phi|z,\eta) \equiv Q_0\Phi(z,\eta) + \mathcal{V}(\Phi|z,\eta) = 0 \tag{2.9}$$

$$\mathcal{V}(\Phi|z,\eta) \equiv \sum_{n\geq 0}\int dz_1\,d\eta_1\ldots dz_{n+2}\,d\eta_{n+2}\,\mathcal{V}^{(n+2)}(z,\eta;z_1,\eta_1,\ldots,z_{n+2},\eta_{n+2})$$
$$\times\,\Phi(z_1,\eta_1)\ldots\Phi(z_{n+2},\eta_{n+2}). \tag{2.10}$$

The linear operator $Q_0$ entering (2.9) is the BRST charge[13] corresponding to the algebra (2.3):

$$Q_0 = \eta^A L_A + \frac{1}{2}(-1)^{\varepsilon_B}\eta^B\eta^C f^A_{CB}\frac{\partial}{\partial\eta^A} \tag{2.11}$$

and $\mathscr{V}(\Phi, z, \eta)$ possesses the properties ($\delta(\eta) \equiv \prod_{A=1}^{\mathscr{N}} \delta(\eta^A)$):

$$\int d\eta\delta(\eta)\mathscr{V}(\Phi|z,\eta) = 0 \tag{2.12a}$$

$$\int d\eta\delta(\eta)\frac{\partial}{\partial\eta^A}\mathscr{V}(\Phi|z,\eta) = V_A(\phi|z). \tag{2.12b}$$

Equations (2.11), (2.12b) ensure that the single equation (2.9) for $\Phi(z,\eta)$ contains the original nonlinear system (2.1):

$$0 = \int d\eta\delta(\eta)\frac{\partial}{\partial\eta^A}Q(\Phi|z,\eta) = L_A\phi(z) + V_A(\phi|z).$$

Let us point out that in each ghost integral first the integration over the fermionic ghosts $c^i$ (2.8) is performed:

$$\int d\eta\mathscr{F}(\eta) = \int d\chi\left[\int dc\mathscr{F}(c,\eta)\right]$$
$$\int dcc^{i_1}\ldots c^{i_M} = \delta_{MN_b}\varepsilon^{i_1\ldots i_{N_b}} \tag{2.13}$$

*Second step*

The new single equation (2.9) must exhibit gauge invariance such that the equations of motion implied by (2.9) for the "nonphysical" part $\tilde{\Phi}(z,\eta)$ of the ghost-haunted field $\Phi(z,\eta)$ (2.7) should have pure gauge solutions, whereas Eq. (2.1) for the original field $\phi(z)$ should be gauge-invariant.

The required gauge invariance has the form:

$$\delta_\Lambda\Phi(z,\eta) = \int dz'\,d\eta'\,\Lambda(z',\eta')\frac{\delta Q(\Phi|z,\eta)}{\delta\Phi(z',\eta')} \tag{2.14}$$

and the gauge invariance of (2.9) under (2.14) implies:[a]

[a] In fact, due to (2.15), the gauge parameter $\Lambda(z,\eta)$ in (2.14) is itself defined only modulo nonlinear transformations

$$\Lambda(z,\eta) \sim \Lambda(z,\eta) + \int dz'\,d\eta'\,\Lambda'(z',\eta')\frac{\delta Q(\Phi|z,\eta)}{\delta\Phi(z',\eta')}.$$

$$\int dz'\, d\eta'\, Q(\Phi|z',\eta')\frac{\delta Q(\Phi|z,\eta)}{\delta\Phi(z',\eta')} = 0\,. \tag{2.15}$$

Inserting in (2.15) the expansion (2.9) for $Q(\Phi|z',\eta')$ one gets:

$$Q_0^2 = 0$$

(i.e. $Q_0$ is a nilpotent operator which is true by construction, see Eqs. (2.11) and (2.6)), and

$$Q_0\mathscr{V}(\Phi|z,\eta) + \int dz'\, d\eta'[Q_0\Phi(z',\eta') + \mathscr{V}(\Phi|z',\eta')]\frac{\delta\mathscr{V}(\Phi|z,\eta)}{\delta\Phi(z',\eta')} = 0\,. \tag{2.16}$$

Therefore, it is natural to call Eq. (2.15) the nonlinear nilpotency condition.

Also note, that due to (2.12a), the original field $\phi(z)$ is inert under the gauge transformation (2.14):

$$\begin{aligned}\delta_\Lambda\phi(z) &= \int d\eta\delta(\eta)\delta_\Lambda\Phi(z,\eta)\\ &= \int dz'\, d\eta'\, \Lambda(z',\eta')\frac{\delta}{\delta\Phi(z',\eta')}\left[\int d\eta\delta(\eta)Q(\Phi|z,\eta)\right] \equiv 0\end{aligned}$$

*Third step*

The action, invariant under (2.14) and producing (2.9) as equation of motion, is now easily derived:

$$S = \int dz\, d\eta\, \hat{H}\Phi(z,\eta)\bar{Q}(\Phi|z,\eta)\,. \tag{2.17}$$

The linear operator $\hat{H}$ is defined to fulfill ("$T$" denotes operator transposition):

$$\begin{aligned}\hat{H}^T &= \hat{H}\\ Q_0^T\hat{H} &= \hat{H}Q_0\,.\end{aligned} \tag{2.17*}$$

A typical form of $\hat{H}$ is

$$\hat{H}\Phi(z,\eta) = R\Phi(\rho_1 z, \rho_2 z)$$

where $R$ is a matrix acting on the vector valued field, $\rho_{1,2}$ are numbers taking the values $\pm 1$, $\pm i$ (cf. (3.36) in the next section). The functional $\bar{Q}(\Phi; z,\eta)$ is defined through the relation:

$$\left[1 + \int dz'\, d\eta'\, \Phi(z',\eta') \frac{\delta}{\delta\Phi(z',\eta')}\right] \bar{Q}(\Phi|z,\eta) = Q(\Phi|z,\eta) \tag{2.18}$$

which simply means:

$$\bar{Q}(\Phi|z,\eta) = \tfrac{1}{2} Q_0 \Phi(z,\eta) + \bar{\mathscr{V}}(\Phi|z,\eta) \tag{2.19}$$

where $\bar{\mathscr{V}}(\Phi|z,\eta)$ is given by a series of the same form as for $\mathscr{V}(\Phi|z,\eta)$ (2.10) with additional multiplication of each $\mathscr{V}^{(n+2)}$ by the factor $(n+3)^{-1}$.

From the fact that $\bar{Q}(\Phi|z,\eta)$ (2.19) enters the action functional (2.17) where one can freely symmetrize the fields $\Phi(z,\eta)$ entering in the various terms, we immediately find that $\bar{Q}$ (2.19) or, equivalently, $Q$ (2.9) should satisfy the antisymmetry condition:

$$\frac{\delta \hat{H} Q(\Phi|z,\eta)}{\delta\Phi(z',\eta')} = -\frac{\delta \hat{H} Q(\Phi|z',\eta')}{\delta\Phi(z,\eta)}. \tag{2.20}$$

Note that the property (2.20) is due to the anticommutativity of the ghost measures (recall $N_b \equiv$ number of $c^i$ = odd)

$$\int d\chi\, dc \int d\chi'\, dc' = -\int d\chi'\, dc' \int d\chi\, dc.$$

Now, it is straightforward to show that the action (2.17) is indeed invariant under the gauge transformation (2.14) provided the nonlinear nilpotency (2.15) and the antisymmetry condition (2.20) hold.

The *Final step* is to derive the explicit expression of $\mathscr{V}(\Phi|z,\eta)$ (2.10) such that (2.15), (2.20) and (2.12) are satisfied. Using (2.13) and (2.11) and taking into account the consistency conditions (2.4) we find:

$$\begin{aligned}\mathscr{V}(\Phi|z,\eta) &= \eta^A V_A(\Phi(\cdot,\eta)|z)\\ &= \sum_{n\geq 0} \int dz_1 \ldots dz_{n+2} \eta^A V_A^{(n+2)}(z;z_1,\ldots,z_{n+2}) \Phi(z_1,\eta)\ldots\Phi(z_{n+2},\eta)\end{aligned} \tag{2.21}$$

where the kernels $V_A^{(n+2)}$ are exactly the same as in (2.2).

Equation (2.21) is the main result in the present general construction since now each object entering the action (2.17) is explicitly expressed in terms of objects entering the original nonlinear system (2.1).

## 3. Superspace Action For $D = 10$ $N = 1$ SYM

It is well-known [6,8,9] that the complete on-shell description of $D = 10$ $N = 1$ SYM theory is given by the Nilsson constraint equations:

$$F^{\alpha\beta} \equiv \frac{1}{g}(\{\nabla^\alpha, \nabla^\beta\} - 2i\nabla\!\!\!/^{\alpha\beta}) = 0. \tag{3.1}$$

We use the standard notations:

$$\begin{aligned} \nabla^\alpha &\equiv D^\alpha + g[A^\alpha, \cdot\} \\ \nabla^\mu &\equiv \partial^\mu + ig[A^\mu, \cdot] \\ D^\alpha &\equiv \frac{\partial}{\partial\theta_\alpha} + i\partial\!\!\!/^{\alpha\beta}\theta_\beta \\ \nabla\!\!\!/^{\alpha\beta} &\equiv \nabla_\mu(\sigma^\mu)^{\alpha\beta} \end{aligned} \tag{3.2}$$

$$F^{\alpha\mu} \equiv D^\alpha A^\mu + i\partial^\mu A^\alpha + g[A^\alpha, A^\mu],$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu].$$

The fundamental fields in the above equations are $A^\mu(x,\theta)$—the vector superfield gauge potential and $A^\alpha(x,\theta)$—the superfield Majorana-Weyl spinor gauge potential. $g$ de-notes the coupling constant.

The Biancchi identities for $\nabla^\alpha$, $\nabla^\mu$ are in fact the consistency conditions for the system (3.1). Multiple application of these identities yields as a consequence of (3.1), the following additional equations for $A^\alpha$, $A^\mu$:[6,8,9]

$$F^{\alpha\mu} - (\sigma^\mu W)^\alpha = 0 \tag{3.3}$$

$$\nabla^\alpha F^{\mu\nu} = ((\sigma^\mu\nabla^\nu - \sigma^\nu\nabla^\mu)W)^\alpha \tag{3.4}$$

$$\nabla^\alpha W_\beta = -\frac{i}{2}(\sigma^{\mu\nu})_\beta{}^\alpha F_{\mu\nu} \tag{3.5}$$

$$\nabla^\mu F_{\mu\nu} = gW\sigma_\nu W \tag{3.6}$$

$$\nabla\!\!\!/ W = 0 \tag{3.7}$$

where $W_\alpha$ is a Majorana-Weyl spinor defined by (3.3).

Now, the system (3.1), (3.3)–(3.7) is a consistent overdetermined system of nonlinear equations for $A^\alpha(x,\theta)$, $A^\mu(x,\theta)$. However, one can easily show that it cannot be written in the form (2.1) with Lorentz-covariant and independent linear operators $L_A$, and, moreover, the condition (2.6) is not satisfied (i.e. the system is of higher rank).

In order to apply our general construction of the action principle, we have to transform (3.1), (3.3)–(3.7) into an equivalent set of nonlinear equations for which the general requirements of independence of $L_A$ and (2.6) are fulfilled. As discussed in Ref. 23, this can be achieved by introducing dependence of $A^\alpha$, $A^\mu$ on additional auxiliary variables $v_\alpha^{\pm 1/2}$, $u_\mu^a$ (to be defined below) and performing a suitable nonlinear field transformation on $A^\alpha(x, \theta, u, v)$, $A^\mu(x, \theta, u, v)$.

The auxiliary variables are introduced as follows:[15–21]

(i) $v_\alpha^{\pm 1/2}$ are two $D = 10$ (left-handed) MW spinors.

(ii) $u_\mu^a$ $(a = 1, \ldots, 8)$ are eight $D = 10$ Lorentz vectors,

which satisfy the kinematical constraints:

$$u_\mu^a u^{b\mu} = C^{ab}$$

$$[v_\alpha^{\pm 1/2} (\sigma^\mu)^{\alpha\beta} v_\beta^{\pm 1/2}] u_\mu^a = 0 \tag{3.8}$$

$$[v_\alpha^{+1/2} (\sigma^\mu)^{\alpha\beta} v_\beta^{+1/2}] [v_\gamma^{-1/2} (\sigma_\mu)^{\gamma\delta} v_\delta^{-1/2}] = -1 .$$

The group $SO(8) \times SO(1, 1)$ acts on $u_\mu^a$, $v_\alpha^{\pm 1/2}$ as an internal group of local rotations where $u_\mu^a$ belongs to any one of the three inequivalent 8-dimensional representations $((v), (s), (c))$ of $SO(8)$, whereas $v_\alpha^{\pm 1/2}$ carry charge $\pm\frac{1}{2}$ under $SO(1, 1)$. $C^{ab}$ denotes the invariant metric tensor in the relevant $SO(8)$ representation space ($C^{ab}$ is the unit matrix for $(v)$ and it is the left (right) chiral charge conjugation matrix for $(s)$, $(c)$, respectively).

Due to the well-known $D = 10$ Fierz identity (see e.g. Ref. 27):

$$(\sigma_\mu)^{\alpha\beta} (\sigma^\mu)^{\gamma\delta} + (\sigma_\mu)^{\beta\gamma} (\sigma^\mu)^{\alpha\delta} + (\sigma_\mu)^{\gamma\alpha} (\sigma^\mu)^{\beta\delta} = 0 \tag{3.9}$$

the composite Lorentz vectors:

$$u_\mu^\pm = v^{\pm 1/2} \sigma_\mu v^{\pm 1/2} \tag{3.10}$$

are identically light-like. Thus the vectors $u_\mu^a$ together with $u_\mu^\pm$ realize the coset space $\dfrac{SO(1, 9)}{SO(8) \times SO(1, 1)}$ (cf. Ref. 12).

Here and below, the following short-hand notations (besides (3.10)) are used:

$$\begin{aligned} A^\pm &\equiv u_\mu^\pm A^\mu \equiv v^{\pm 1/2} \not{A} v^{\pm 1/2} \\ A^a \equiv u_\mu^a A^\mu ; &\qquad \sigma^{a_1 \ldots a_n} \equiv u_{\mu_1}^{a_1} \ldots u_{\mu_n}^{a_n} \sigma^{[\mu_1} \ldots \sigma^{\mu_n]} \end{aligned} \tag{3.11}$$

for any Lorentz vector $A^\mu$. Let us particularly stress that $A^\pm$, $A^a$ are Lorentz scalars and they should not be confused with the vector components of $A^\mu$ which appear in the noncovariant light-cone formalism.

Now, we regard the superfields $A^\alpha$, $A^\mu$ in (3.1), (3.3)–(3.7) as harmonic superfields on the extended superspace $z = (x^\mu, \theta_\alpha, u_\mu^a, v_\alpha^{\pm 1/2})$.[16] As harmonic superfields, $A^\alpha(z)$, $A^\mu(z)$ are not arbitrary functions of $(u, v)$. Rather, they belong to the class of functions defined by an expansion in monomials of $u^a_\mu$, $v^{\pm 1/2}_\alpha$ where all internal SO(8) × SO(1, 1) indices are saturated among the $u$'s and $v$'s. The coefficients of these monomials in the expansion are then, ordinary superfields which do not carry any SO(8) × SO(1, 1) indices. Consequently, the harmonic superfields, identically fulfill:

$$(D^{ab}, D^{-+})\begin{bmatrix} A^\alpha(z) \\ A^\mu(z) \end{bmatrix} = 0 \tag{3.12}$$

where the harmonic differential operators

$$D^{ab} = u^a_\mu \frac{\partial}{\partial u_{\mu b}} - u^b_\mu \frac{\partial}{\partial u_{\mu a}} + \frac{1}{2}\left( v^{+1/2}\sigma^{ab}\frac{\partial}{\partial v^{+1/2}} + v^{-1/2}\sigma^{ab}\frac{\partial}{\partial v^{-1/2}} \right) \tag{3.13}$$

$$D^{-+} = \frac{1}{2}\left( v^{+1/2}_\alpha \frac{\partial}{\partial v^{+1/2}_\alpha} - v^{-1/2}_\alpha \frac{\partial}{\partial v^{-1/2}_\alpha} \right) \tag{3.14}$$

represent an SO(8) × SO(1, 1) algebra.

To ensure the on-shell independence of $A^\alpha$, $A^\mu$ on $(u, v)$, we add the following harmonic differential equations:

$$D^{\pm a}\begin{bmatrix} A^\alpha(z) \\ A^\mu(z) \end{bmatrix} = 0 \tag{3.15}$$

where:

$$D^{\pm a} = u^\pm_\mu \frac{\partial}{\partial u_{\mu a}} + \frac{1}{2} v^{\mp 1/2}\sigma^\pm \sigma^a \frac{\partial}{\partial v^{\mp 1/2}}\,. \tag{3.16}$$

The complete set of harmonic differential invariant operators $D^{ab}$, $D^{-+}$, $D^{\pm a}$ preserve the kinematical constraints (3.8) and form an SO(1, 9) algebra under commutation.

As it was discussed in detail in Ref. 16 (see also Ref. 23), the 45 equations (3.12), (3.15) plus the 53 kinematical constraints (3.8) are sufficient to yield the on-shell independence of the harmonic superfields on all 112 auxiliary $u$, $v$ variables.[b]

Furthermore, we introduce the following nonlinear field transformation:

$$\begin{bmatrix} A^\alpha(z) \\ A^\mu(z) \end{bmatrix} \to \phi(z) = \begin{bmatrix} Y^{+1/2a}(z) \\ B^a(z) \end{bmatrix} \tag{3.17}$$

$$Y^{+1/2a}(z) = \frac{i}{2}(v^{+1/2}\sigma^a\sigma^-)_\alpha \partial^+ \left[ \Omega^{-1}(z)A^\alpha(z)\Omega(z) + \frac{1}{g}\Omega^{-1}(z)D^\alpha\Omega(z) \right] \tag{3.18}$$

[b] Therefore the statement in a recent paper by Kallosh and Rahmanov[26] claiming "nonunitarity" of the present formalism is incorrect.

$$B^a(z) = u^a_\mu\left[\Omega^{-1}(z)A^\mu(z)\Omega(z) - \frac{i}{g}\Omega^{-1}(z)\partial^\mu\Omega(z)\right] \tag{3.19}$$

(here $\partial^+ \equiv u^+_\mu\partial^\mu$).

The superfield $\Omega(z)$ in (3.18), (3.19) takes values in the YM gauge group and it is a functional of $A^\mu(z)$, solving the equation $(u^+_\mu\nabla^\mu)\Omega = 0$:

$$\Omega(z) = P\exp\left\{-ig\int^{x^-} u^+_\mu A^\mu\big(x(y^-;u,v),\theta,u,v\big)\,dy^-\right\}$$
$$x^- \equiv u^-_\mu x^\mu, \qquad x^\mu(y^-;u,v) \equiv (\eta^{\mu\nu} + u^{+\mu}u^{-\nu})x_\nu - u^{+\mu}y^-. \tag{3.20}$$

In Ref. 23 it is explicitly shown that the system (3.1), (3.3)–(3.7), (3.15) of nonlinear equations for $A^\alpha(z)$, $A^\mu(z)$ is equivalent to the following system of nonlinear equations for $\phi(z)$ (3.17)–(3.20):

$$(-\partial^2)\phi(z) + V_0(\phi|z) = 0 \tag{3.21}$$

$$\hat{D}^\alpha\phi(z) + V^\alpha_1(\phi|z) = 0 \tag{3.22}$$

$$\hat{D}^{-a}\phi(z) + V^{-a}_2(\phi|z) = 0 \tag{3.23}$$

$$D^{+a}\phi(z) = 0 \tag{3.24}$$

with the notations as explained below.

The linear operators in (3.22)–(3.23) acting on $\phi(z)$ (3.17) are defined as ($\partial^a \equiv u^a_\mu\partial^\mu$):

$$\hat{D}^\alpha\phi \equiv \begin{bmatrix} D^\alpha Y^{+1/2a} - i(\partial\!\!\!/\sigma^b\sigma^a v^{+1/2})^\alpha B_b \\ D^\alpha B^a - \frac{1}{\partial^+}(\partial\!\!\!/\sigma^a\sigma^b v^{+1/2})^\alpha Y^{+1/2}_b \end{bmatrix} \tag{3.25}$$

$$\hat{D}^{-a}\phi \equiv \begin{bmatrix} \left(D^{-a} - \frac{1}{2}\frac{\partial^a}{\partial^+}\right)Y^{+1/2b} - \frac{\partial_c}{\partial^+}(S^{ac})^b{}_d Y^{+1/2d} \\ D^{-a}B^b - \frac{\partial_c}{\partial^+}(V^{ac})^b{}_d B^d \end{bmatrix} = 0. \tag{3.26}$$

In (3.26) the $8 \times 8$ matrices $S^{ab}$, $V^{ab}$ denote the generators of the Lorentz invariant harmonic ($s$) and ($v$) representation of SO(8):[21]

$$(S^{ab})_{cd} \equiv \tfrac{1}{2}v^{+1/2}\sigma_c\sigma^{ab}\sigma^-\sigma_d v^{+1/2}$$
$$(V^{ab})^{cd} \equiv C^{ac}C^{bd} - C^{ad}C^{bc}. \tag{3.27}$$

Since $\phi(z)$ (3.17) is a 16-component spinor, so are the nonlinear terms in (3.21)–(3.23)

$$V_0(\phi|z) = \begin{bmatrix} [V_0(\phi|z)]^{(Y)a} \\ [V_0(\phi|z)]^{(B)a} \end{bmatrix}$$

and, similarly for $V_1^\alpha$, $V_2^{-a}$. The explicit form of $V_0(\phi|z)$ reads:

$$\begin{aligned}[V_0(\phi|z)]^{(Y)a} \equiv & -ig\left(\partial^b[B_b, Y^{+1/2a}] + [B_b, \nabla'^b Y^{+1/2a}] + \left[\partial^+ B_b, \frac{1}{\partial^+}\nabla'^b Y^{+1/2a}\right]\right) \\ & + 2ig\partial^+\left[\frac{1}{(\partial^+)^2}(\nabla'_c\partial^+ B^c - g\{Y^{+1/2c}, Y_c^{+1/2}\}), Y^{+1/2a}\right] \\ & - 2ig\left[\partial^+ B_c, (S^{cd})^{ab}\frac{1}{\partial^+}\nabla'_d Y_b^{+1/2}\right] - ig[F'_{cd}, (S^{cd})^{ab} Y_b^{+1/2}]; \end{aligned} \tag{3.28}$$

$$\begin{aligned}[V_0(\phi|z)]^{(B)a} \equiv & \ \partial_b(\nabla'^a B^b) - \partial^+\nabla'^a\frac{1}{(\partial^+)^2}(\nabla'_c\partial^+ B^c - g\{Y^{+1/2c}, Y_c^{+1/2}\}) \\ & + ig\left[\frac{1}{(\partial^+)^2}(\nabla'_c\partial^+ B^c - g\{Y^{+1/2c}, Y_c^{+1/2}\}), \partial^+ B^a\right] + ig[B_b, F'^{ab}] \\ & + 2g(S^{ac})_{bd}\left\{\frac{1}{\partial^+}\nabla'_c Y^{+1/2b}, Y^{+1/2d}\right\} - g\left\{\frac{1}{\partial^+}\nabla'^a Y^{+1/2c}, Y_c^{+1/2}\right\} \end{aligned} \tag{3.29}$$

with the following notations:

$$\nabla'^a \equiv \partial^a + ig[B^a, \ ]$$

$$F'^{ab} \equiv \partial^a B^b - \partial^b B^a + ig[B^a, B^b]$$

and $(S^{ab})_{cd}$ as in (3.27).

For $V_1^\alpha(\phi|z)$ we have:

$$\begin{aligned}[V_1^\alpha(\phi|z)]^{(Y)a} \equiv & \ 2ig(v^{+1/2}\sigma^b)\left\{\frac{1}{\partial^+}Y_b^{+1/2}, Y^{+1/2a}\right\} - \frac{g}{2}(v^{+1/2}\sigma^a\sigma^{bc})^\alpha[B_b, B_c] \\ & - ig(v^{+1/2}\sigma^a)^\alpha\frac{1}{\partial^+}(\{Y^{+1/2c}, Y_c^{+1/2}\} - i[B_c, \partial^+ B^c]) \end{aligned} \tag{3.30}$$

$$[V_1^\alpha(\phi|z)]^{(B)a} \equiv 2ig(v^{+1/2}\sigma^b)^\alpha\frac{1}{\partial^+}\left[\frac{1}{\partial^+}Y_b^{+1/2}, \partial^+ B^a\right] - ig(v^{+1/2}\sigma^b\sigma^a\sigma^c)^\alpha\frac{1}{\partial^+}[B_c, Y_b^{+1/2}] \tag{3.31}$$

and for $V_2^{-a}(\phi|z)$ we have:

$$[V_2^{-a}(\phi|z)]^{(Y)b} \equiv ig\left[Y^{+1/2b}, \frac{1}{\partial^+}B^a\right] + i\frac{g}{2}\frac{1}{\partial^+}[Y^{+1/2b}, B^a] + ig(S^{ac})^{bd}\frac{1}{\partial^+}[Y_d^{+1/2}, B_c] \tag{3.32}$$

$$[V_2^{-a}(\phi|z)]^{(B)b} \equiv gC^{ab}\frac{1}{(\partial^+)^2}(\{Y^{+1/2c}, Y_c^{+1/2}\} - i[B_c, \partial^+ B^c]) + ig\left[B^b, \frac{1}{\partial^+}B^a\right]. \tag{3.33}$$

Finally, the linear operators in (3.21)–(3.24) are independent and form a closed algebra (2.3).[21,23]

The corresponding BRST charge is first rank, i.e. condition (2.6) holds. Its explicit expression in matrix form reads:

$$Q_0 = \begin{bmatrix} Q_0^{(YY)}C^{ab} & [Q_0^{(YB)}]^{ab} \\ [Q_0^{(BY)}]^{ab} & Q_0^{(BB)}C^{ab} \end{bmatrix} \tag{3.34}$$

$$Q_0^{(YY)} = c(-\partial^2) + \chi_\alpha D^\alpha - (2i\partial^+)^{-1}(\chi\sigma^+\chi)\frac{\partial}{\partial c} + i\eta_a^- D^{+a}$$

$$+ i\eta_a^+\left[D^{-a} - \frac{1}{2}\frac{\partial^a}{\partial^+} - \frac{\partial_b}{\partial^+}S^{ab} - \frac{1}{2}(\partial^+)^{-2}\eta_b^+ S^{ab}\frac{\partial}{\partial c}\right]$$

$$Q_0^{(BB)} = c(-\partial^2) + \chi_\alpha D^\alpha - (2i\partial^+)^{-1}(\chi\sigma^+\chi)\frac{\partial}{\partial c} + i\eta_a^- D^{+a}$$

$$+ i\eta_a^+\left[D^{-a} - \frac{\partial_b}{\partial^+}V^{ab} - \frac{1}{2}(\partial^+)^{-2}\eta_b^+ V^{ab}\frac{\partial}{\partial c}\right]$$

$$[Q_0^{(YB)}]^{ab} = -i(\chi\partial\!\!\!/\sigma^b\sigma^a v^{+1/2}) + \frac{i\eta_c^+}{\sqrt{2}(2\partial^+)}(\chi\sigma^+\sigma^{cd}v^{-1/2})(U\gamma_d)^{ab}\frac{\partial}{\partial c}$$

$$[Q_0^{(BY)}]^{ab} = -\frac{1}{\partial^+}(\chi\partial\!\!\!/\sigma^a\sigma^b v^{+1/2}) - \sqrt{2}\frac{\eta_c^+}{(2\partial^+)^2}(\chi\sigma^+\sigma^{cd}v^{-1/2})(\tilde{\gamma}_d U)^{ab}\frac{\partial}{\partial c}$$

where

$$(\gamma^a)_{bc} \equiv \sqrt{2}v^{+1/2}\sigma_b\sigma^a\sigma_c v^{-1/2}, \qquad (\tilde{\gamma}^a)_{bc} \equiv \sqrt{2}v^{-1/2}\sigma_b\sigma^a\sigma_c v^{+1/2}$$

are the Lorentz invariant SO(8) $\sigma$ matrices and the matrix $U$ is:[21]

$$[U]^{ab} \equiv 2(v^{+1/2}\sigma^a\sigma^c\sigma^b v^{-1/2})(v^{+1/2}\sigma_c v^{-1/2}).$$

The ghosts ($\eta^A$) correspond to the linear operators ($L_A$) (3.21)–(3.26) according to the table:

$$\begin{bmatrix} L_A & | & -\partial^2 & \hat{D}^\alpha & D^{+a} & \hat{D}^{-a} \\ \eta^A & | & c & \chi_\alpha & \eta^{-a} & \eta^{+a} \end{bmatrix}. \tag{3.35}$$

The operator $\hat{H}$ fulfilling the conditions (2.17*) was found in Ref. 23 to be:

$$\hat{H} = \begin{bmatrix} \frac{1}{2}(K_1 + K_1^T)\frac{1}{\partial^+} & 0 \\ 0 & \frac{1}{2}(K_2 + K_2^T) \end{bmatrix} \tag{3.36}$$

where $K_{1,2}$ acts on the arguments of the corresponding functions as follows:

$$K_1 \colon v_\alpha^{\pm 1/2} \to \pm i v_\alpha^{\pm 1/2}$$

$$c \to -c$$

$$\eta^{\pm a} \to -\eta^{\pm a},$$

$$K_2 \colon v_\alpha^{\pm 1/2} \to \pm i v_\alpha^{\pm 1/2}$$

$$\chi_\alpha \to -\chi_\alpha .$$

Thus, the nonlinear system (3.21)–(3.24) for $\phi(z)$ (3.17)–(3.20) constitutes the complete on-shell description of $D = 10$ $N = 1$ SYM. Its crucial feature is that it meets all the requirements necessary for the construction of an action according to our general scheme in Sec. 2. Therefore, the superspace action in terms of unconstrained (off-shell) superfields of $D = 10$ $N = 1$ SYM is given by formulae (2.17), (2.19), (2.21) by substituting there (3.34)–(3.36), (3.28)–(3.33) and (3.17)–(3.20).

As a final remark, let us stress that the superspace action (2.17) is also manifestly invariant under the superspace YM gauge transformation of the ghost-haunted superfields $\mathscr{A}^\mu(z,\eta)$, $\mathscr{A}^\alpha(z,\eta)$:

$$\begin{aligned} \mathscr{A}^\mu(z,\eta) &\to (\mathscr{A}^\omega)^\mu(z,\eta) = \omega^{-1}(z,\eta)\left(\mathscr{A}^\mu(z,\eta) - \frac{i}{g}\partial^\mu\right)\omega(z,\eta) \\ \mathscr{A}^\alpha(z,\eta) &\to (\mathscr{A}^\omega)^\alpha(z,\eta) = \omega^{-1}(z,\eta)\left(\mathscr{A}^\alpha(z,\eta) + \frac{1}{g}D^\alpha\right)\omega(z,\eta). \end{aligned} \tag{3.37}$$

This is because the action (2.17) depends on $\mathscr{A}^\alpha$, $\mathscr{A}^\mu$ only through the ghost-haunted superfield expression

$$\Phi(z,\eta) \equiv \begin{bmatrix} \frac{i}{2}(v^{+1/2}\sigma^a\sigma^-)_\alpha \partial^+ \left(\Omega^{-1}(z,\eta)\mathscr{A}^\alpha(z,\eta)\Omega(z,\eta) + \frac{1}{g}\Omega^{-1}(z,\eta)D^\alpha\Omega(z,\eta)\right) \\ u_\mu^a\left(\Omega^{-1}(z,\eta)\mathscr{A}^\mu(z,\eta)\Omega(z,\eta) - \frac{i}{g}\Omega^{-1}(z,\eta)\partial^\mu\Omega(z,\eta)\right) \end{bmatrix} \tag{3.38}$$

which is itself invariant under (3.37). Here $\Omega$ is a functional on the ghost-haunted superfield $\mathscr{A}^\mu$. Its explicit form is given by (3.20) where one substitutes $\Omega(z)$ by $\Omega(z, \eta)$ and $A^\mu(z)$ by $\mathscr{A}^\mu(z, \eta)$.

The ghost-haunted superfield $\Phi(z, \eta)$ (3.38) contains the original superfield $\phi(z)$ (3.17)–(3.20), i.e. the ghost-haunted super-potentials $\mathscr{A}^\mu(z, \eta)$, $\mathscr{A}^\alpha(z, \eta)$ contain the original SYM superfields $A^\mu(z)$, $A^\alpha(z)$ according to the general formula (2.7) (recall $z = (x^\mu, \theta_\alpha, u^a_\mu, v^{\pm 1/2}_\alpha)$ and $\eta^A$ from (3.35)).

## Conclusion

In the present paper we have formulated a covariant action principle for arbitrary consistent overdetermined systems of nonlinear field equations with the help of the BFV-BRST ghost formalism.

This allowed us to solve the longstanding problem of finding a superspace action in terms of (off-shell) unconstrained superfields for $D = 10$ $N = 1$ SYM.

Our action principle resembles the Siegel-Zwiebach-Witten-Neveu-West construction of (super)string field actions but does not involve the peculiarities (star-products, Chern-Simons terms) specific for the RNS (super)string field theory context. In particular, our $D = 10$ SYM action contains both cubic and quartic interaction terms.

The most ambitious task is to apply the present formalism for constructing a manifestly super-Poincare covariant field theory of the GS superstring employing the super-Poincare covariant first quantization procedure of Refs. 18, 20 and 21.

## Acknowledgments

Two of us (E.N., S.P.) are deeply indebted to the Einstein Center for Theoretical Physics and Prof. Y. Frishmann for their cordial hospitality at the Weizmann Institute of Science, Rehovot.

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